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Reasoning About Obligations in *Obligationes*: A Formal Approach

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Abstract

Despite the appearance of ‘obligation’ in their name, medieval obligational disputations between an Opponent and a Respondent seem to many to be unrelated to deontic logic. However, given that some of the example disputations found in medieval texts involve Respondent reasoning about his obligations within the context of the disputation, it is clear that some sort of deontic reasoning is involved. In this paper, we explain how the reasoning differs from that in ordinary basic deontic logic, and define dynamic epistemic semantics within which the medieval obligations can be expressed and the examples evaluated. Obligations in this framework are history-based and closely connected to action, thus allowing for comparisons with, e.g., the knowledge-based obligations of Pacuit, Parikh, and Cogan, and *stit*-theory. The contributions of this paper are twofold: The introduction of a new type of obligation into the deontic logic family, and an explanation of the precise deontic concepts involved in *obligationes*.

Keywords: deontic logic, dynamic epistemic logic, obligation, *obligationes*, *stit*

1 Introduction

Deontic logicians who are interested in the history of their field may upon first introduction to the medieval genre of *disputationes de obligationibus* think they have found their ancestor: For what else could treatises on “disputations concerning obligations” be about other than reasoning about obligation and permission, i.e., deontic logic? Closer inspection of these disputations, however, may lead the deontic logician to a place of puzzlement, for the example disputations which can be found in the treatises often have little or nothing to do with obligation, permission, commitment, or any of the related notions which make up the core of deontic logic. On an initial survey, it is unclear what the obligation involved in these disputations is or how they are related to reasoning about deontic principles in general—if at all. In fact, many contemporary scholars of *obligationes* explicitly disavow any connection between obligational

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disputations and deontic logic [18,19,25].² Those who do take seriously the notion of ‘obligation’ in terms of discursive commitments primarily focus on how these obligations function at the meta-level of the disputation and disputational norms, [8,9,10,13,14,23]. To date, little work has been done on the formal nature of the obligations involved in *obligationes*.

There are two participants in an obligational disputation, Opponent and Respondent.³ Opponent begins by putting forth (‘positing’) a proposition which Respondent either admits or does not admit. (If he does not admit it, then no disputation begins.) Propositions in the disputation are divided into two types, relevant (*pertinens*) and irrelevant (*impertinens*). A proposition is relevant if either it or its negation is a logical consequence of the set of propositions which have already been admitted (in the initial round of the disputation) or conceded (in a later round of the disputation) along with the negations of those denied, and it is irrelevant otherwise. The relevant propositions are typically further divided into those which are ‘relevant and following’ (*pertinens sequens*) and those which are ‘relevant and contradictory’ (*pertinens repugnans*). The typical rule is that any relevant following proposition must be conceded, any relevant contradictory proposition must be denied; and any irrelevant proposition must be conceded if it is known to be true, denied if known to be false, and doubted if neither (where doubting is taken as a neutral action).

These disputations, so simple to describe, have nevertheless been a matter of contention amongst contemporary scholars—and not just because it is unclear what (if anything) the relation is between them and deontic logic. Another puzzling feature of these disputations is that they often appear to be empty of content [18]; the examples that occur in the medieval treatises do not involve any substantive doctrinal issues. Instead, the propositions which appear in many examples have a feeling of genericity to them: When Opponent puts forward that Socrates is white or that Plato is black, or that Socrates and Plato have the same color, it is clear that what is at stake is not the substantive question of what color two ancient Greek philosophers were. Likewise, when Respondent denies that a human being is a donkey or concedes that God exists, he is not making a point about biology or theology. Instead, these propositions are functioning as arbitrary contingent, impossible, and necessary propositions, whose content is less important than their modal status.

But not all of the disputations are like this. In this paper we examine some examples where the propositions put forward are not about Socrates, Plato,

² [12] does not explicitly reject a connection, but does so implicitly by not mentioning *obligationes* at all.

³ Because we do not have the space to go into the complexities of the historical situation, we vastly oversimplify here. What we say here is true of the type of *obligatio* called *positio* ‘positing’, and of the rules for *positio* given by Walter Burley in his treatise on *obligationes*, c.1302. Readers interested in the historical information are directed to [21,22] for further information. In this paper, we present only those portions of the medieval theory which are necessary to motivate the proposed formalization.

donkey-humans, or God. Instead, the propositions Opponent posits have to do with Respondent's own responses, both the responses that he does make and the responses he ought to make. This brings deontic reasoning directly into the disputational framework, forcing Respondent to reason about his obligations explicitly within the object language of the disputation.

2 *Obligationes* and deontic logic

In the brief description above of the rules that Respondent must follow when responding to Opponent's propositions, it is clear from the outset that there are three different places where deontic concepts such as "must" or "ought to" come in—one for each of the different types of actions Respondent can make. What are these obligations rooted in? They arise when Respondent admits the *positum* and thus binds himself to play by the rules. That is, they are generated by his actions. This is true for the remainder of the disputation, in that it is his actions which give rise to further obligations which he must abide by. In this, the basic type of obligation that is involved in *obligationes* differs relevantly from those in ordinary deontic logic. In the dominant approach to deontic logic [2, p. 23], the operator 'it is obligatory that' attaches to propositions, and no agent is specified, because it is expressed in the passive voice. (Such approaches violate the Restricted Complement Thesis, which will appear below in §4.) When $O\varphi$ is asserted in a deontic logic context, there is no indication of whose responsibility it is to see to it that φ is the case, or what actions can or should be taken in order to meet the obligation.⁴ In contrast, in an *obligatio*, the obligation exists between an agent and an action.

Given this, there is a sense in which those who disavow any connection between *obligationes* and deontic logic are right; it is certainly true no direct comparison can be done. However, it is still possible to ask how these two types of obligation, albeit different, compare to each other, and the answer sheds interesting light on the question of what the obligation in an *obligatio* is. Boh briefly considered this question in [5]. He offers the following formalization of "the three most general constitutive rules of (the main type of) an obligational disputation called *positio*" [5, p. 112]:⁵

- $\forall\varphi(K_a\varphi \rightarrow O_a C\varphi)$
- $\forall\varphi(K_a\neg\varphi \rightarrow O_a N\varphi)$
- $\forall\varphi((\neg K_a\varphi \wedge \neg K_a\neg\varphi) \rightarrow O_a D\varphi)$

According to Boh, "Rule (1) is read: for any proposition which is or might be put forward in a disputation in which the person a takes part, if a knows that φ , it is obligatory that he grants it. . . 'O', of course, represents the deontic operator of obligation" [5, p. 112]. The other rules are to be understood analogously. Unfortunately, inspection of the medieval texts will quickly show that these

⁴ Our use of "see to it that" here is not accidental. We discuss the relationship between our approach and stit-theory in §6.

⁵ We have slightly adapted his notation to be consistent with the notation used in the current paper (see Def. 4.1), as well as corrected errors in parentheses in the second two formulas.

are untenable as correct formulations of the principle duties of Respondent in *positio*. Virtually every author writing on this type of obligation points out explicitly that it is only worthwhile to pursue a *positio* when the *positum* is (known to be) false. Thus, Respondent upon admitting such a *positum* will immediately violate (2), by admitting and conceding a proposition which is known to be false.⁶

Let us take a step back and ask a more general question: In what way are the obligations in *obligationes* like the obligations in deontic logic? For a given deontic logical language \mathcal{L}_d , the minimal deontic logic is axiomatized by the following schemata [24, p. 274]:

- (i) All axiom schemata and rules of classical logic over \mathcal{L}_d .
- (ii) $O\varphi \wedge O\psi \rightarrow O(\varphi \wedge \psi)$
- (iii) $\neg O\neg\top$

and the rule of inference:

$$\frac{\varphi \Rightarrow \psi}{O\varphi \Rightarrow O\psi}$$

If we interpret the O above as “ought to concede”, then the second schema says that if Respondent ought to concede two conjuncts individually in their own right, then he also ought to concede their conjunction. This is true in the obligational setting only in certain cases. There are three possibilities: φ and ψ are both relevant and following, φ and ψ are both irrelevant and known to be true, and one of φ or ψ is relevant and following and the other is irrelevant and known to be true. In the first two cases, if Respondent ought to concede φ and he ought to concede ψ , then he also ought to concede $\varphi \wedge \psi$. However, the third case fails. Suppose that Respondent ought to concede φ because it is relevant and following, even though in reality it is known to be false. Suppose that Respondent ought to concede ψ because it is irrelevant and known to be true. Then consider $\varphi \wedge \psi$. The conjunction is neither relevant and following nor relevant and contradictory, since in either case, ψ would then also be relevant. Thus, $\varphi \wedge \psi$ is irrelevant, and since φ is known to be false, the conjunction is known to be false as well. As a result, not only is Respondent not obligated to concede the conjunction, he is obligated to deny it. A similar story shows that the rule of inference also fails. Suppose that $\varphi \rightarrow \psi$ and Respondent ought to concede φ . If he ought to concede φ because it is relevant and following, then it is true he also ought to concede ψ , because whatever φ follows from, ψ follows from as well, if $\varphi \rightarrow \psi$. However, suppose φ is irrelevant, but known to be true. If ψ is relevant in its own right, then it is true that Respondent ought to concede ψ , independently of his obligations towards φ . But if ψ is irrelevant, it is possible that $\varphi \rightarrow \psi$ but Respondent does not know that ψ is true. In such a case, his obligation to ψ is to doubt it, not to concede it. (In such a

⁶ Burley gives an example which violates Boh’s reconstruction: “It should be said that ‘My hand is not closed’ must be denied even though it is true. And that is because it is incompatible, since its opposite was previously granted” (earlier in the disputation) [7, 3.30].

way, logical omniscience does not arise in *obligationes*, as noted by Uckelman [21, p. 20].)

However, a modified version of this schema does hold: If Respondent *has in fact* conceded φ , and likewise he has also conceded ψ , then it is true that he ought to concede their conjunction: His previous actions have changed his obligations with respect to $\varphi \wedge \psi$. This turns out to be the crucial insight into understanding how the obligations in *obligationes* differ from the ones in ordinary deontic logic. To state the (perhaps) obvious: Actions change commitments. A person's commitments in a given situation are influenced both by what he does and what he knows. These obligations are, to borrow a term from game-theory, *history-dependent*. (We discuss this further in §6.)

3 An example

Before we turn to the technical details, let us consider a concrete example, due to Walter Burley. After he lists the rules for *positio*, Burley considers various objections to these rules in the forms of sophisms, or logically problematic sentences, where it appears that Respondent has no consistent way of responding in a disputation beginning with such a sophism. The third objection to the first rule⁷ runs as follows:

Objection 3.1 *Let this be posited: ‘You are in Rome or that you are in Rome must be granted’. Next, let ‘That you are in Rome must be granted’ be proposed. This is false and irrelevant; therefore it must be denied. Next, let ‘That you are in Rome follows from the positum and the opposite of a proposition already correctly denied’ be proposed. This is necessary, because this conditional is necessary: ‘If either you are in Rome, or that you are in Rome must be granted, but that you are in Rome is not to be granted, then you are in Rome.’ Once this has been granted—‘That you are in Rome follows from the positum and the opposite of a proposition already correctly denied’—let this be proposed: ‘That you are in Rome must be granted’. If you grant this, you have granted and denied the same thing; therefore [you have responded badly]. If you deny it, the disputation is over; you have denied what follows according to a rule. Because if the rule is good, this follows: ‘That you are in Rome follows from the positum and the opposite of a proposition already correctly denied; therefore that you are in Rome must be granted’ [7, 3.21].*

This is presented as an objection to the rule because—if it is a correct description of how to proceed in such a case—it conflicts with the general principle that a Respondent who follows the rules correctly will never be forced to concede a contradiction. We briefly sketch in an informal fashion why one might think the objection correctly describes how Respondent should proceed. The

⁷ The first rule is: “Everything that follows from the positum must be granted. Everything that follows from the positum either together with an already granted proposition (or propositions), or together with the opposite of a proposition (or the opposites of propositions) already correctly denied and known to be such, must be granted” [7, 3.15]. Different translators use ‘granted’ and ‘conceded’ to translate Latin *concedendum*.

positum takes the form of a disjunction, $p \vee q$, with the assumption being that both disjuncts are false.⁸ Next, one disjunct, q , is put forward. Neither p nor q follow from $p \vee q$ alone in the absence of other information (and we have been provided with no such additional information), and q is, by assumption, false. Thus, q should be denied. If q is denied, then p follows from this denial along with the earlier concession of $p \vee q$, by simple application of disjunctive syllogism. But $p \vee q$ is the *positum*, and if it is granted that p follows from this along with the opposite of something correctly denied, namely q , this is tantamount to saying that p ought to be granted—and this fact itself is q , which was previously denied.

Three points should be noted. First, we reiterate what we said above about the apparent contentless nature of the disputations. Whether Respondent is in Rome is a matter of little import.⁹ This example should be understood not as an example of a specific dialogue, but of a template into which any contingent falsehood whatever can be substituted in for “You are in Rome”.

Second, we equated the “opposite of something correctly denied” with that proposition’s contradictory negation. This is not a problematic equation, but nevertheless it is one that should be noted explicitly.

Third, when we equated the *positum* with a simple disjunction of the form $p \vee q$, this overlooked the internal structure of the two disjuncts, in particular the fact that p occurs in the second disjunct as well. The only way that we can take this internal structure into account is if we can express the notion of obligation embedded in ‘must be granted’ and ‘must be denied’ within the object language itself, not only in the metalanguage. Extending the ordinary object language of *obligationes* so that it can handle cases like this one will shed light on the type of obligation involved, and also how one can use the *obligationes* framework in order to carry out (a certain limited type of) deontic reasoning. Because we must first have the sophisticated framework in place before we can understand how the example works, we defer discussion of Burley’s solution to the objection to §5.

4 An abstract model for *obligationes*

In this section we define an abstract model, built on the framework defined in [21] for Burley-style *obligationes*, within which specific examples, such as the one above, can be analysed. The language is standard multi-agent dynamic epistemic language, with two agents O(pponent) and R(espondent) and three actions:

Definition 4.1 Let φ_n be a proposition put forward by O. R’s available ac-

⁸ Though it should be noted that the assumption is that Respondent is in fact not in Rome; Burley wrote the treatise in Oxford, and thus he and his potential opponents and respondents were located there as well.

⁹ See footnote 8.

tions are C (for *concedendo*), N (for *negando*), and D (for *dubitando*):

$$\begin{aligned} C \varphi_n &:= [\varphi_n]^\top \\ N \varphi_n &:= [\neg \varphi_n]^\top \\ D \varphi_n &:= [\top]^\top \end{aligned}$$

Well-formed formulas of this language are defined in the usual way, and these formulas are interpreted on epistemic Kripke models, again as is standard. We give only the semantics for the three actions. For an epistemic Kripke model \mathfrak{M} , let

$$\mathfrak{M} \upharpoonright \varphi = \langle W^{\mathfrak{M}, \varphi}, \{\sim_a^{\mathfrak{M}, \varphi} : a \in \mathcal{A}\}, V^{\mathfrak{M}, \varphi} \rangle,$$

where $W^{\mathfrak{M}, \varphi} := \{w \in W : \mathfrak{M}, w \models \varphi\}$, and the relations and valuation functions are just restrictions of the originals. For an ordered set of propositions Γ_n , let $\mathfrak{M} \upharpoonright \Gamma_n = \mathfrak{M} \upharpoonright \gamma_0 \upharpoonright \dots \upharpoonright \gamma_n$, that is, $\mathfrak{M} \upharpoonright \Gamma_n$ is the result of the sequential restriction of \mathfrak{M} by the elements of Γ_n . Then:

Definition 4.2 $\mathfrak{M}, w \models [\varphi]\psi$ iff $\forall v \in \mathfrak{M} \upharpoonright \varphi, v \models \psi$.

When Respondent announces “I concede φ ”, the model is reduced to only those worlds where φ is true, and when he announces “I deny φ ”, the model is reduced to only those worlds where φ is false. The action of doubt is strongly agnostic: If Respondent is in doubt about a particular proposition, and he announces this $\mathfrak{M} \upharpoonright \top = \mathfrak{M}$, i.e., nothing changes. Given these semantics, the actions defined above can be understood as tests for consistency. If Respondent’s actions are correct, then after his announcement there will be at least one world left in the model. This captures the fact, noted by Burley himself, that if Respondent responds according to the rules (which we define formally below), then he will never be led into inconsistency, that is, into an empty model.

From an abstract perspective, an obligational disputation can be seen as a pair of sequences, one of propositions which are put forward by Opponent and the other is the actions of Respondent¹⁰, along with a rule which indicates the specific type of disputation the *obligatio* is.

Definition 4.3 An *obligatio* is a quadruple $\mathfrak{O} = \langle \Theta, R, \Gamma^R, \Gamma \rangle$ where

- Θ is a sequence of propositions, such that $\theta_0 \in \Theta$ is the initial proposition and $\theta_n \in \Theta$ is the proposition put forward by O at round n .
- $R : \Theta \times \mathbb{N} \rightarrow \text{Act}$ is a function determining R’s correct response to each element of Θ . We write $R(\theta_n)$ for $R(\theta, n)$ to simplify notation.
- Γ^R is a sequence of elements from Act , formed by the *correct* response of R to each element in Θ , as given by R . That is:

$$\begin{aligned} \Gamma_0^R &= \langle R(\theta_0) \rangle \\ \Gamma_n^R &= \langle \gamma_0, \dots, \gamma_{n-1}, R(\theta_n) \rangle \end{aligned}$$

Whether Γ^R is unique depends on R .

¹⁰ We could also view it as a single sequence, formed of these two sequences interleaved.

- Γ is a sequence of elements from Act , formed by R 's actual responses to each element of Θ .

Note that there are no constraints on Γ : In principle, Respondent is free to respond in any fashion that he likes (so long as it is one of the actions ‘concede’, ‘deny’, or ‘doubt’). In practice, however, if Respondent’s responses are not directed to the immediately preceding proposition of Opponent, then it makes more sense to think of him not actually participating in an *obligatio* disputation, rather than participating but failing wholly to play by the rules. We do not add this as a formal constraint, but simply note that we do not consider cases where Respondent acts in such a mulish, recalcitrant fashion.

We formalize Burley’s rules for *positio*, presented informally in §1, as follows:

Definition 4.4 For a model \mathfrak{M} and formula $\theta_0 \in \Theta$:

$$R^{\text{Bur}}(\theta_0) = \begin{cases} C \theta_0 & \text{iff } \mathfrak{M}, w \models \langle \theta_0 \rangle \top \\ N \theta_0 & \text{otherwise} \end{cases}$$

For $\theta_n \in \Theta, n > 0$:

$$\begin{array}{ll} \text{If } \mathfrak{M} \upharpoonright \Gamma_{n-1} \models \theta_n: & R^{\text{Bur}}(\theta_n) = C \theta_n \\ \text{If } \mathfrak{M} \upharpoonright \Gamma_{n-1} \models \neg \theta_n: & R^{\text{Bur}}(\theta_n) = N \theta_n \\ \text{Otherwise, let } w^* \text{ be the actual world, and:} & \\ \text{If } \mathfrak{M}, w^* \models K_R \theta_n: & R^{\text{Bur}}(\theta_n) = C \theta_n \\ \text{If } \mathfrak{M}, w^* \models K_R \neg \theta_n: & R^{\text{Bur}}(\theta_n) = N \theta_n \\ \text{If } \mathfrak{M}, w^* \models \neg(K_R \theta \vee K_R \neg \theta_n): & R^{\text{Bur}}(\theta_n) = D \theta_n \end{array}$$

If Respondent denies θ_0 , then no *obligatio* begins. We represent this by having him deny the *positum*, which has the effect of ‘canceling’ the model and not allowing any further progress. Admitting the *positum*, which triggers the start of the disputation, is equivalent in effect, if not exactly in action, to conceding it. For an *obligatio* according to these rules, Γ^R will always be uniquely defined.

These rules provide the grounding for the obligations in the disputation. Respondent is obliged to follow the rules, or risk the charge of having responded badly, and hence ‘losing’ the disputation. Thus, Respondent’s obligations in a *positio* according to the rule defined in Def. 4.4 are:

Definition 4.5 In an *obligatio* \mathfrak{O} at round n

- R ought to concede φ iff either $\mathfrak{M} \upharpoonright \Gamma_{n-1} \models \varphi$; or $\mathfrak{M} \upharpoonright \Gamma_{n-1} \not\models \varphi$, $\mathfrak{M} \upharpoonright \Gamma_{n-1} \not\models \neg \varphi$, and $\mathfrak{M}, w^* \models K_R \varphi$.
- R ought to deny φ iff either $\mathfrak{M} \upharpoonright \Gamma_{n-1} \models \neg \varphi$; or $\mathfrak{M} \upharpoonright \Gamma_{n-1} \not\models \varphi$, $\mathfrak{M} \upharpoonright \Gamma_{n-1} \not\models \neg \varphi$, and $\mathfrak{M}, w^* \models K_R \neg \varphi$.
- R ought to doubt φ iff $\mathfrak{M} \upharpoonright \Gamma_{n-1} \not\models \varphi$, $\mathfrak{M} \upharpoonright \Gamma_{n-1} \not\models \neg \varphi$, and $\mathfrak{M}, w^* \models \neg(K_R \varphi \vee K_R \neg \varphi)$.

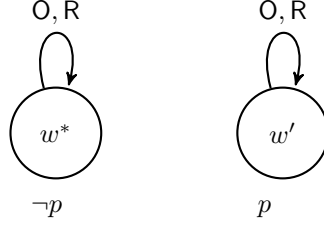


Fig. 1.

These obligations provide us with the truth conditions for an object-language obligation operator O_a ‘ a ought to’ which can be applied to actions. That is, our O operator satisfies the Restricted Complement Thesis [3, p. 787], which requires that “the deontic constructions such as obligation, prohibition, and permission must take agentives as their complements”. While only actions can be obligatory (and not propositions), for any proposition, it is in principle possible that Respondent be obliged to conceded, deny, or doubt that proposition.

Definition 4.6 Fix an *obligatio* \mathfrak{O} and model \mathfrak{M} .

$$\begin{aligned}
 \mathfrak{M}|\Gamma_{n-1}, w \models O_R C \varphi_n & \text{ iff } \text{either } \mathfrak{M}|\Gamma_{n-1} \models \varphi \\
 & \text{ or } \mathfrak{M}|\Gamma_{n-1} \not\models \varphi, \mathfrak{M}|\Gamma_{n-1} \not\models \neg\varphi \\
 & \text{ and } \mathfrak{M}, w^* \models K_R \varphi \\
 \mathfrak{M}|\Gamma_{n-1}, w \models O_R N \varphi_n & \text{ iff } \text{either } \mathfrak{M}|\Gamma_{n-1} \models \neg\varphi, \\
 & \text{ or } \mathfrak{M}|\Gamma_{n-1} \not\models \varphi, \mathfrak{M}|\Gamma_{n-1} \not\models \neg\varphi \\
 & \text{ and } \mathfrak{M}, w^* \models K_R \neg\varphi \\
 \mathfrak{M}|\Gamma_{n-1}, w \models O_R D \varphi_n & \text{ iff } \mathfrak{M}|\Gamma_{n-1} \not\models \varphi, \mathfrak{M}|\Gamma_{n-1} \not\models \neg\varphi \\
 & \text{ and } \mathfrak{M}, w^* \models \neg(K_R \varphi \vee K_R \neg\varphi)
 \end{aligned}$$

Note that the obligations are global: They do not depend on the world of evaluation.

We are now in a position to formalize and evaluate the *positio* given in §3. Let $p := \text{‘R is in Rome’}$. Then, the *positum* $\theta_0 = p \vee O_R C p$, and the rest of Θ for the *positio* is as follows:

$$\begin{aligned}
 \theta_1 &:= O_R C p \\
 \theta_2 &:= (\theta_0 \wedge \neg\theta_1) \rightarrow p \\
 \theta_3 &:= O_R C p
 \end{aligned}$$

Consider the model in Figure 1, with two worlds, one where Respondent is in Rome and one where he is not; the latter is the actual world, and both participants know this.

Proposition 4.7 *The positum $\theta_0 = p \vee O_R C p$ is false at the actual world because both disjuncts are false.*

Proof. (1) $\mathfrak{M}, w^* \models \neg p$. (2) $\mathfrak{M} \not\models p$, because of w^* , and $\mathfrak{M} \not\models \neg p$, because of w' . $\mathfrak{M}, w^* \models \neg p$, so $\mathfrak{M}, w^* \models \neg K_R p$. Hence, $\mathfrak{M} \not\models O_R C p$. \square

However, the *positum* is not inconsistent (since $w' \models p$), so Respondent is correct in admitting it, and the resulting model $\mathfrak{M}|\theta_0$ is displayed in Figure 2.

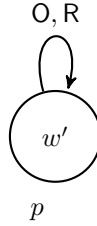


Fig. 2. $\mathfrak{M}|\theta_0$

Opponent then posits $O_R C p$. Immediately we can see where the objection Burley considers has gone wrong. The objection says that $O_R C p$ “is false and irrelevant; therefore it must be denied”. However, once $p \vee O_R C p$ has been admitted, $O_R C p$ is no longer irrelevant. The only situations where Respondent ought to concede p are cases where either p is true in all remaining worlds of the model *or* where p is known at the actual world. Because we start from the assumption that p is false at the actual world, it cannot be known there, which means that after conceding θ_0 , the only worlds that remain are worlds which were retained because they made p true. Hence, whatever the initial model was like, after conceding $p \vee O_R C p$, $O_R C p$ becomes relevant. Thus, the objection fails at round 1.

5 Burley’s solution

It is of course interesting that we are able to use our formalization to identify a problem with the objection. But the only way this will go beyond merely *interesting* is if the problem that we have identified is the same problem that Burley identifies; and if not, does it tell us something about the problem he identifies? This is one test of the adequacy of the framework. It turns out that Burley solves the objection differently than we did in the previous section, in the following way:

Solution 5.1 (To Objection 3.1) *This must be denied: ‘That you are in Rome follows from the positum and the opposite of a proposition correctly denied’; it is not necessary either. Even if it is necessary that from the posited disjunction together with the opposite of one disjunct it follows that you are in Rome, it is nonetheless not necessary that the disjunction be posited [7, 3.22].*

Few modern commentators have discussed either the objection or this response to it, and those who have find it puzzling. Stump says that this solution to the problem “looks bizarre”, and that:

On the face of it, then, Burley is saying that $[\theta_2]$ is to be denied because it is not necessary, and his reason for claiming that $[\theta_2]$ is not necessary is that one of the premisses it is derived from, namely, $[\theta_0]$ is not necessary [20, p. 324].

But Stump is correct in noting that “there is no obligational rule to the effect that we must deny any propositions which are not necessary” [20, p. 324], and this is clearly not what Burley is arguing here. One way to understand what he is saying is to look at another objection and response that he presents, since the second response makes much the same point:

Objection 5.2 *Let this be posited: ‘Nothing is posited to you.’ Next, let ‘Everything that follows from the positum must be granted’ be proposed. This must be granted because it is a rule. Next, let ‘Something follows from the positum’ be proposed. This follows and therefore must be granted. Next, let ‘Something is posited’ be proposed. If you grant this, you grant the opposite of the positum; therefore, [you have responded] badly. If you deny it, you deny something that follows, because this follows: ‘Something follows from the positum; therefore something is posited’ [7, 3.17].*

Solution 5.3 (To Objection 5.2) *One says that this must be denied: ‘Everything that follows from the positum must be granted’; it is not necessary either. But this is necessary: If something follows from the positum, it must be granted [7, 3.19].*

In both solutions, Burley identifies a proposition in the disputation which Respondent had conceded but which in fact he ought to have denied, and justifies this fact by an appeal to a lack of necessity. Stump offers a different interpretation of the solution to Objection 3.1 on which “an unintelligible solution [turns] into an intelligible red herring” [20, p. 325]. When Burley says “‘That you are in Rome’ follows from the *positum* and the opposite of a proposition correctly denied” must be denied, Stump argues that Burley takes an “extreme” [20, p. 327] stance with respect to the phrase ‘the *positum*’, treating it as ambiguous because it may refer to the *positum* in the disputation at hand, or it may refer to any other potential *positum*. Thus, when Burley says “it is not necessary that the disjunct be posited either”, he is pointing out that if another proposition had been the *positum*—which is perfectly reasonable—then none of the other propositions put forward would follow. This interpretation is bolstered by Burley’s solution to yet another objection (which we do not discuss), in which he makes a distinction between “You have denied something that follows from the *positum*, therefore [you have responded] badly” and “You have denied something that follows from what was posited to you, therefore [you have responded] badly” [7, 3.20]. The only way that these can differ is if ‘the *positum*’ can be interpreted more generically than ‘what was in fact posited to you’.¹¹

There are two ways to react to this interpretation. On the one hand, calling this a red herring may not be incorrect, since on this interpretation the solution doesn’t appear to address the issue we identified in the previous section, namely, that given the relationship between admittance and concession, once

¹¹It is worth remembering here that Latin does not have definite or indefinite articles, and that occurrences of ‘the *positum*’ are better read as simply ‘*positum*’ without any article.

the *positum* of Objection 3.1 has been admitted, the second disjunct becomes relevant.¹² On the other hand, calling it a mere red herring is dismissive, implying that what Burley is demonstrating here is in some sense beside the point. But this is not the case. In fact, what Burley is pointing out in each of the three solutions is a genuine feature of obligational disputations—namely, that until the *positum* is admitted, no obligation exists. The rules themselves that Respondent can bind himself to follow are not themselves necessary. They are not encoded in the models, and while Respondent ought to follow the rules, he is not forced to, because sometimes he can make mistakes. What Burley is pointing out is the fact that Respondent is not under obligation until he admits the *positum*; only after that does it become necessary that he grant whatever it is that does in fact follow from what was posited to him initially. It is precisely that the obligations can be violated—that they are weaker than necessity—which our approach illustrates so clearly. Thus, while the primary problem with the objections that our framework identifies is not the one Burley identifies, our model nevertheless sheds light on the solutions he does give, and provides an explanation which has hitherto been elusive.

6 Comparison with related work

In §2 we briefly explained how the axioms and inference rule of the minimal deontic logic fail to obtain in the context of obligational reasoning. In this section, we compare what we have introduced above to two other well-known approaches which combine action and obligation, knowledge-based obligations and *stit*-theory.

6.1 Knowledge-based obligations

In [15], Pacuit, Parikh, and Cogan (PPC) introduce a semantics for knowledge-based obligations which model the interaction between knowledge and obligation in history-based models such as those of [16,17]. These obligations bear a strong resemblance to the obligations in *obligationes*, and in fact much of what we developed above was inspired by the PPC approach. Nevertheless, there are some important differences, which we highlight.

PPC models are composed of sets of events indexed to each agent, and a set of all global possible histories of these event-sets. Obligations in these models are expressed by the introduction of a value function for histories which assigns a real number to each global infinite history such that higher-valued histories are considered ‘better’ than lower-valued ones [15, p. 321]. An agent is then obliged to perform a certain action at a certain time if any maximal extension of the agent’s local history is one where that action is considered ‘good’ [15, Def. 4.2].

The first difference is conceptual. The goal of [15] was to explain how the creation of new knowledge can engender obligations for an agent that he did not previously have—that is what makes the obligations of PPC *knowledge-*

¹²Stump ultimately recognizes this fact [20, p. 326].

based. In our case, we also want to represent a type of dependency but it is based on action rather than knowledge. We could say that the obligations in *obligationes* are knowledge-based, but the knowledge is rooted in the actions of Respondent. Thus, the dependency on knowledge is only derivative; the dependency on action is primary.

The second is structural, relating to the shape of the models. History-based models are Kripke frames built from a set of events which are organized into sequences, linear histories which themselves make up a branching structure. The models distinguish between local (agent-indexed) histories and global (agent-indifferent) histories, and there is a global clock which is used to keep the histories synchronous. In our framework, there is no history parameter which is internal to the model. There is no temporal reasoning or actions which happen within a given model; all of the action happens at the level of transitioning from one model to another, via the public concession, denial, and doubting of propositions by Respondent. These transitions are tracked at the level of the *obligatio*, with the parameter Γ , rather than at the level of the model that the *obligatio* is evaluated against. If we consider the sequence of models that a particular Γ gives rise to as forming the history of the disputation in which Respondent's obligations are grounded, then it is clear that we do not have to distinguish between local histories and global histories, because both participants are aware of every move made in the disputation; there is no uncertainty as to whether, e.g., Opponent put forward a certain proposition, or whether Respondent conceded it. By keeping the 'histories' separate from the models, we are easily able to explain what grounds the high value of Γ_R , which by definition (Def. 4.3) is the history which follows the rules R .

Additionally, doing this obviates the need for a global clock: Each move in the disputation functions as a 'clock tick', in one sense. The fact that the 'clock' is external to the model reflects one of the structural properties of *obligationes* which we haven't otherwise discussed in this paper, namely, the often-stated rule that "all responses must be directed to the same instant" [7, 3.84]. That is, the determination of the correct action for irrelevant propositions should be done with respect to a single, fixed moment—in our set-up, the actual world in the initial model—so as to prevent Respondent from being forced to concede "Opponent is sitting" put forward while Opponent is in fact sitting, but then having to deny it when Opponent is no longer sitting.

6.2 stit-theory

Since its introduction in [4], the 'seeing to it that' approach to agency has proven a rich source of tools for reasoning about agency and action. It is no surprise that many people have used different flavors of stit-operators to define various explicit obligation operators. We survey only a few of these approaches here and do not make any claim to completeness.

Similar to the PPC approach, in branching stit frames obligations are generally expressed by the addition of an *Ought* function which maps each moment m to a subset of $H(m)$, the set of histories containing m . More fine-grained

notions of obligation can be expressed by placing valuations on the histories, allowing histories to be compared to each other as being better or worse. Deontic operators defined in these ways lie between historical necessity and historical possibility [11, p. 616], that is, if it is necessary for Respondent to concede φ , then he ought to concede φ ; and if he ought to concede φ , then it is possible for him to concede φ . In another approach ([6], adapted from [1]), “an agent is obliged to do something if and only if by not doing it, it performs a violation” [6, p. 55]. This informal description of obligation is consistent with the conception of obligation in *obligationes*, whereby Respondent is required to do that which will keep him from responding badly. Formally, the obligation operator is defined as follows:

$$O[axstit]\varphi := \Box(\neg[axstit]\varphi \rightarrow [axstit]V)$$

where V is a violation.

Belnap sees stit-theory as “a powerful alternative to two different programs”. The first program we already saw in §2, and relates a deontic operator to a proposition. The second relates deontic operators to actions. Belnap criticizes this approach because it “does not offer us a logical point of view from which it is easy and natural to see that obligation, etc., can in fact make at least *subordinate* reference to declaratives” [2, p. 23]. He says that stit :

make[s] it easy to see that obligation must take an imperative, and also easy to see the important truth that any declarative whatsoever can give rise to an imperative [2, p. 23].

The same can be said of the actions and obligations in *obligationes*: Since Opponent can put forward any sentence whatsoever, not only can any declarative give rise to an imperative, any (well-formed within the language) imperative can as well! Not only can Opponent put forward φ , giving rise to $O_R X\varphi$ for $X \in \{C, N, D\}$, but he can also put forward $O_R X\varphi$, giving rise to $O_R X(O_R X\varphi)$.

Nevertheless, conceptually there is a large difference between obligations in stit-theory and obligations in *obligationes*, and that is that stit-theory, despite being about imperatives rather than declaratives, is still oriented towards the resulting proposition, not towards the action which causes the result. To put it simply, $Rstit\varphi$ encodes an arbitrary action with a definite outcome—the sentence says nothing about what action R must take in order to see to it that φ is true, only that φ ’s being true must result, if the stit sentence is true. In an *obligatio*, on the other hand, $C\varphi$ encodes an arbitrary outcome with a definite action—this sentence says nothing about what the result is, only what action Respondent must take. Given his preceding actions, at each round there is a definite action which Respondent is obligated to perform, regardless of the consequences performing that action might have.

7 Conclusion

Historically, many people have disavowed any connection between the medieval genre of *obligationes* and deontic logic in general. What we have shown here is that on the contrary, the obligational setting provides an interesting framework for reasoning about dialectical obligations in an agent-directed rather than a proposition-directed way. The approach has features in common with both knowledge-based obligations of PPC and with various types of obligations developed in conjunction with *stit*, but differs from both in important ways by focusing on the specific actions which give rise to the obligations. Thus, even if it were stripped of its medieval trappings, the present framework provides an interesting starting point for further exciting investigations.

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